GET READY

Before building equations of circles, it's worth sharpening two tools used constantly on the coordinate plane: the **distance formula** and the **midpoint formula**.

Midpoint Formula

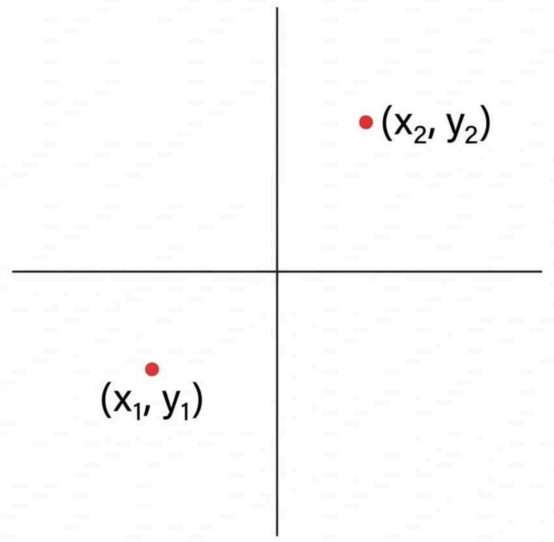
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Finds the exact point halfway between two coordinates.

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Finds the exact length of the segment connecting two points (x_1, y_1) and (x_2, y_2).



Warm-Up

A segment has endpoints: $(1, 2)$ and $(9, 8)$. What is the midpoint and distance between the points?

Midpoint: $M = \left(\frac{1+9}{2}, \frac{2+8}{2}\right) = (5, 5)$

Distance: $d = \sqrt{(9-1)^2 + (8-2)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$

Given points $(-3, 4)$ and $(5, -2)$, what is the distance between them?

Select one

1. 10
2. 100
3. 2
4. 14

Correct

Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(5 - (-3))^2 + (-2 - 4)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

Incorrect

100 is the value under the radical ($8^2 + (-6)^2 = 64 + 36 = 100$). Remember to take the square root of 100 as the final step of the distance formula: $\sqrt{100} = 10$.

LEARN

Every point on a circle sits the exact same distance (the radius) from one fixed point, the center. That single idea is baked directly into the circle's standard equation.

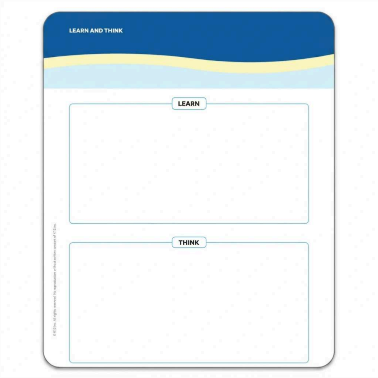
The Standard Equation

$(x-h)^2+(y-k)^2=r^2$

This equation is the distance formula in disguise: it states that the distance from any boundary point (x, y) to the center (h, k) is always r.

- (x, y): any point sitting on the circle's boundary
- (h, k): the center. **Flip the sign** of the number inside each parenthesis to read it off directly
- r^2: the constant on the right side. **Take the square root** to get the actual radius r

You can use the *Learn* part of the page in the workbook to show your thinking or take notes.



Worked Example

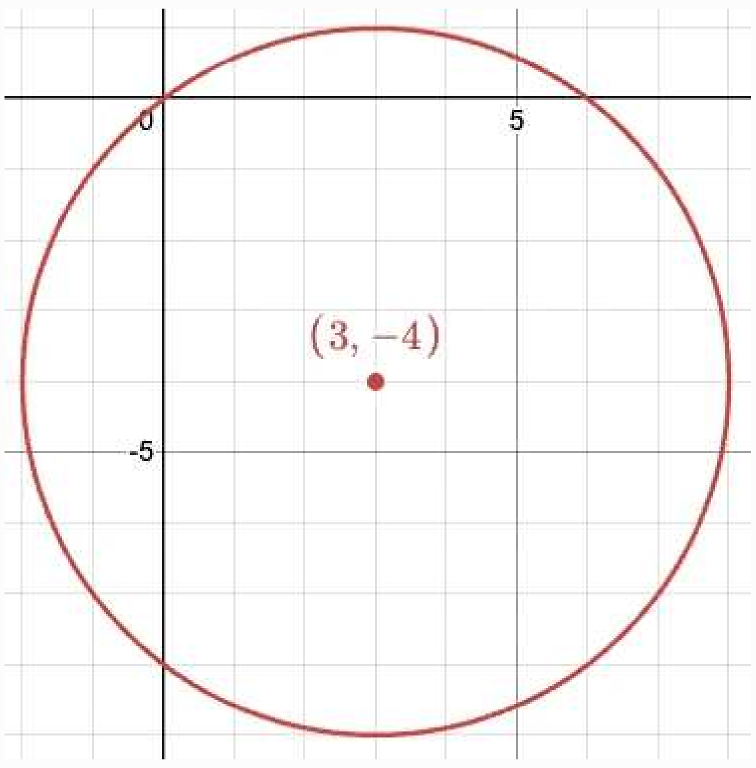
Here is the equation of a circle: $(x-3)^2+(y+4)^2=25$. Use the circle's equation to find its center and radius.

Step 1 (center): Rewrite $(y+4)$ as $(y-(-4))$. Flipping signs gives center $(3,-4)$.

Step 2 (radius): $r^2=25$, so $r=\sqrt{25}=5$

Result: center $(3,-4)$, radius 5

Now examine the same circle graphed on a coordinate plane. Use the circle's graph to find its center and radius.



1. locate the center point on the grid: $(3,-4)$
2. count grid units horizontally or vertically from the center to the boundary to find the radius: 5

> **Match the Graph Back to the Equation**
>
> The center's coordinates go inside the parentheses (signs-flipped), and the radius squared goes on the right.

Worked Example

Sometimes a circle's center is known along with just one point on its edge, not the radius directly. Center $(1,2)$, and the point $(4,6)$ lies on the circle. Find the radius.

Step 1: The radius is the distance from the center to any point on the circle, so apply the distance formula between $(1,2)$ and $(4,6)$.

Step 2: $r=\sqrt{(4-1)^2+(6-2)^2}=\sqrt{9+16}=\sqrt{25}=5$

Result: radius 5, so the equation is $(x-1)^2+(y-2)^2=25$

Here are three other scenarios for solving circle problems:

- finding a radius from a center and one point on the circle
- finding a center and radius from diameter endpoints
- finding a point on a circle given its center and radius

Expand each section to explore the worked example.

Center + One Point on the Circle → Radius +

Sometimes a circle's center is known along with just one point on its edge, not the radius directly. Center $(1, 2)$, and the point $(4, 6)$ lies on the circle. Find the radius.

Step 1: The radius is the distance from the center to any point on the circle, so apply the distance formula between $(1, 2)$ and $(4, 6)$.

Step 2: $r = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Result: radius 5, so the equation is $(x-1)^2 + (y-2)^2 = 25$

Diameter Endpoints → Center and Radius +

Sometimes the center and radius aren't given directly: only the two endpoints of a diameter are provided. The full equation can still be built using familiar tools. A circle has a diameter with endpoints $(-2, 4)$ and $(6, -2)$. Find the center and radius of the circle and write the equation of the circle.

Step 1 (Center): Apply the midpoint formula to the two endpoints. $M = \left(\frac{-2+6}{2}, \frac{4+(-2)}{2}\right) = (2, 1)$.

Step 2 (Radius): Apply the distance formula between the center and either endpoint. $r = \sqrt{(6-2)^2 + (-2-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$.

Result: center $(2, 1)$, radius 5, so the equation is $(x-2)^2 + (y-1)^2 = 25$

Center + Radius → A Point on the Circle +

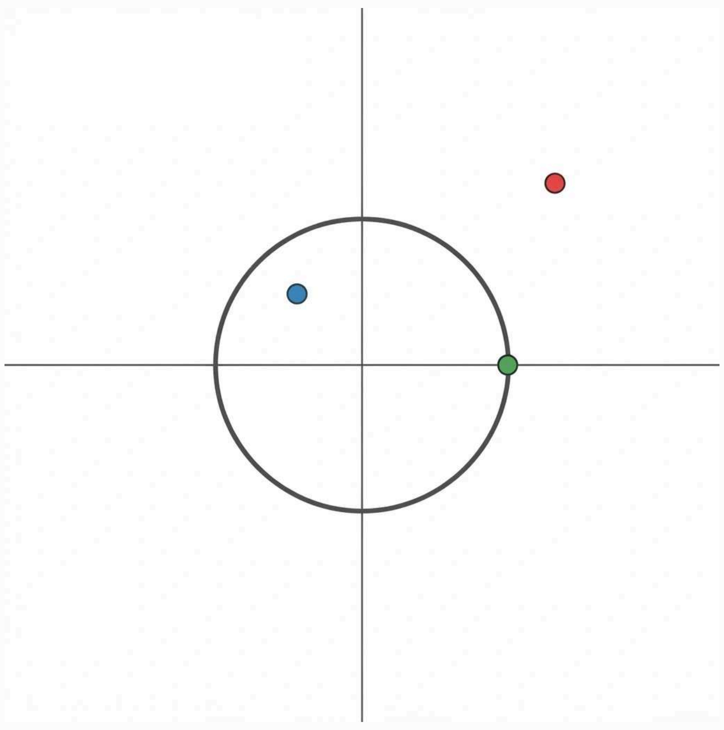
The reverse is also possible: given a center and a radius, a point on the circle can be found without being told one directly. Center $(2, -3)$, radius 4. Find one point on the circle.

Step 1: Move exactly the radius length away from the center in any direction. Moving 4 units to the right along the x-axis: $(2+4, -3) = (6, -3)$.

Step 2 (check): Substitute $(6, -3)$ into the equation with center $(2, -3)$: $(6-2)^2 + (-3-(-3))^2 = 16 + 0 = 16 = 4^2$. ✓

Result: $(6, -3)$ lies on the circle $(x-2)^2 + (y+3)^2 = 16$

Testing a Point Against a Circle



Once an equation is known, it can reveal whether any coordinate point is inside, on, or outside the circle. Substitute the point's (x, y) into the left side and compare the result to r^2.

- equal to r^2 → the point is on the circle
- less than r^2 → the point is inside the circle
- greater than r^2 → the point is outside the circle

Given the circle $(x-1)^2 + (y-2)^2 = 16$, confirm that the point $(2, 3)$ lies inside the circle.

Step 1: Substitute the point into the left side of the equation: $(2-1)^2 + (3-2)^2 = 1 + 1 = 2$.

Step 2 (Compare): Compare the result to $r^2 = 16$. Since $2 < 16$, the point is inside the circle.

THINK

Why does the standard equation of a circle use subtraction, $(x - h)$, even when the center's coordinate is positive? What would change if the signs were **not** flipped when reading the center?

Type answer

If a circle's equation shows x^2 with no visible subtraction next to it, what does that reveal about the center's x-coordinate, and why?

Type answer

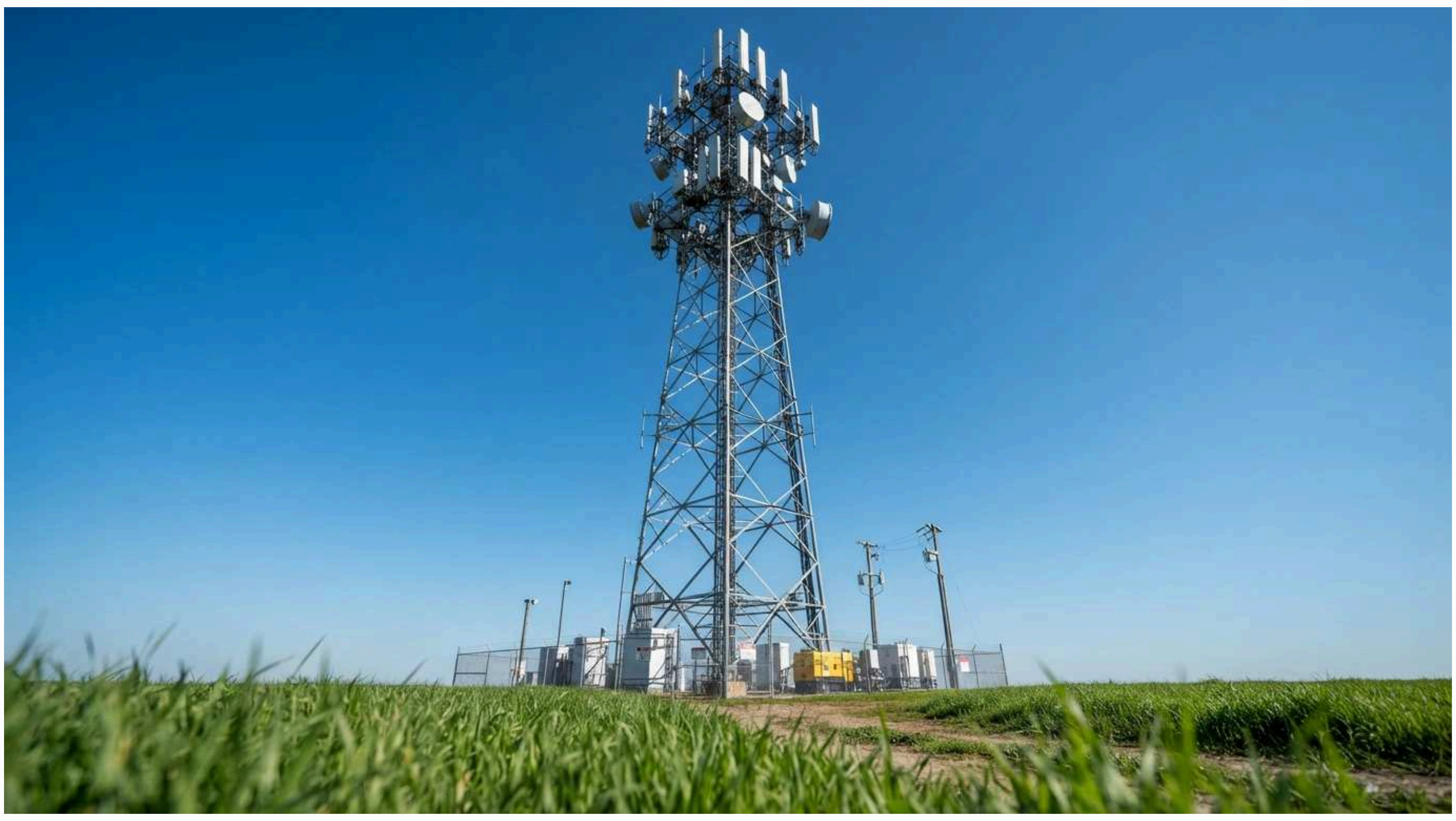
Two circles share the same radius but have different centers. How would their equations differ, and how would their graphs differ?

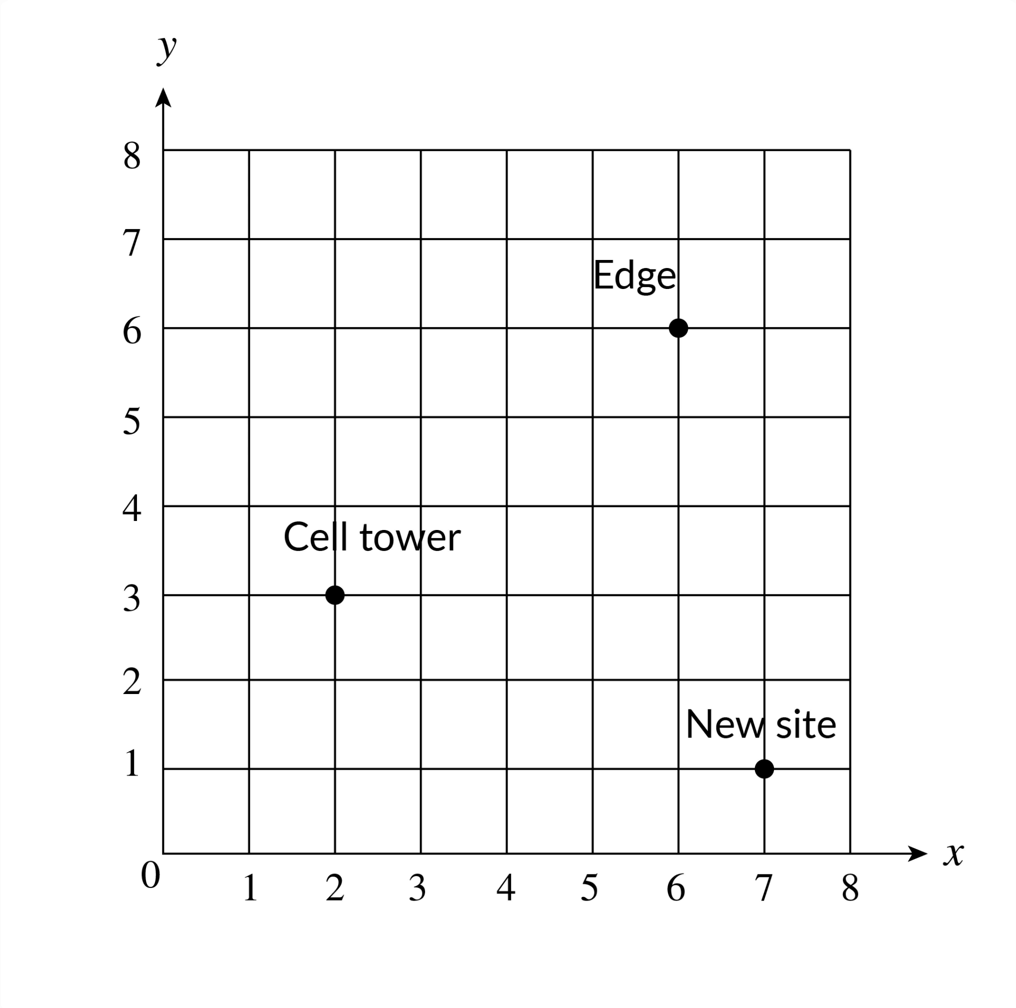
Type answer

A classmate claims the radius of $(x - 2)^2 + (y - 1)^2 = 18$ is 18. What is their mistake, and what is the actual radius?

Type answer

A cell provider is mapping the exact edge of a tower's signal range on a coordinate grid. Circle knowledge can help analyze it.





The Scenario

- Tower center: $(2, 3)$
- A known point on the exact edge of coverage: $(6, 6)$
- Proposed new site: $(7, 1)$

How can the center $(2, 3)$ and the edge point $(6, 6)$ be used to calculate the exact radius of the coverage zone? Show the reasoning and find the radius.

Type answer

Once the radius is known, how is the full standard equation of the boundary circle written?

Type answer

Is the proposed site $(7, 1)$ inside, outside, or exactly on the boundary? Use substitution to justify the answer with numbers, not just a guess.

Type answer

Suppose the New Site could be moved closer to or farther from the tower along the same path. As it moves, the substituted value $(x-2)^2 + (y-3)^2$ changes too. At what exact value does the classification flip between 'inside' and 'outside', and why?

Type answer

TRY IT

Pull every skill together: match equations to centers, read a radius straight from a standard-form equation, work out a radius from diameter endpoints while catching a common error, verify points by substitution, and prove a point lies on a given circle.

Match **Each** Equation to Its Center

$(0, 2)$	$(x-3)^2+(y+4)^2=25$
$(3, -4)$	$(x+1)^2+(y-5)^2=16$
$(-1, 5)$	$x^2+(y-2)^2=9$
$(-2, -3)$	$(x-6)^2+y^2=49$
$(6, 0)$	$(x+2)^2+(y+3)^2=4$

What is the radius of the circle given by the equation $(x+3)^2+(y-5)^2=64$? Do not use the diameter-endpoints method; read the radius directly from the equation.

Type answer

A circle has diameter endpoints at $(-4, 1)$ and $(2, 7)$. A classmate calculates the radius as 6, using only the horizontal distance between the endpoints. Find the correct center and exact radius, and explain the specific error in the classmate's reasoning.

Type answer

Classify **each** point relative to the circle $(x-1)^2+(y-2)^2=16$

outside the circle	$(5, 2)$
inside the circle	$(3, 4)$
outside the circle	$(0, 0)$
on the circle	$(1, 7)$
inside the circle	$(4, -1)$

A circle has center $(4, -1)$ and radius 6. The point $(10, -1)$ lies on this circle. Identify the coordinates of **one other point** that also lies on the circle, and show the substitution that proves it satisfies the equation.

Type answer